A Story of M Theory and Quantum Mechanics

Kimyeong Lee
KIAS

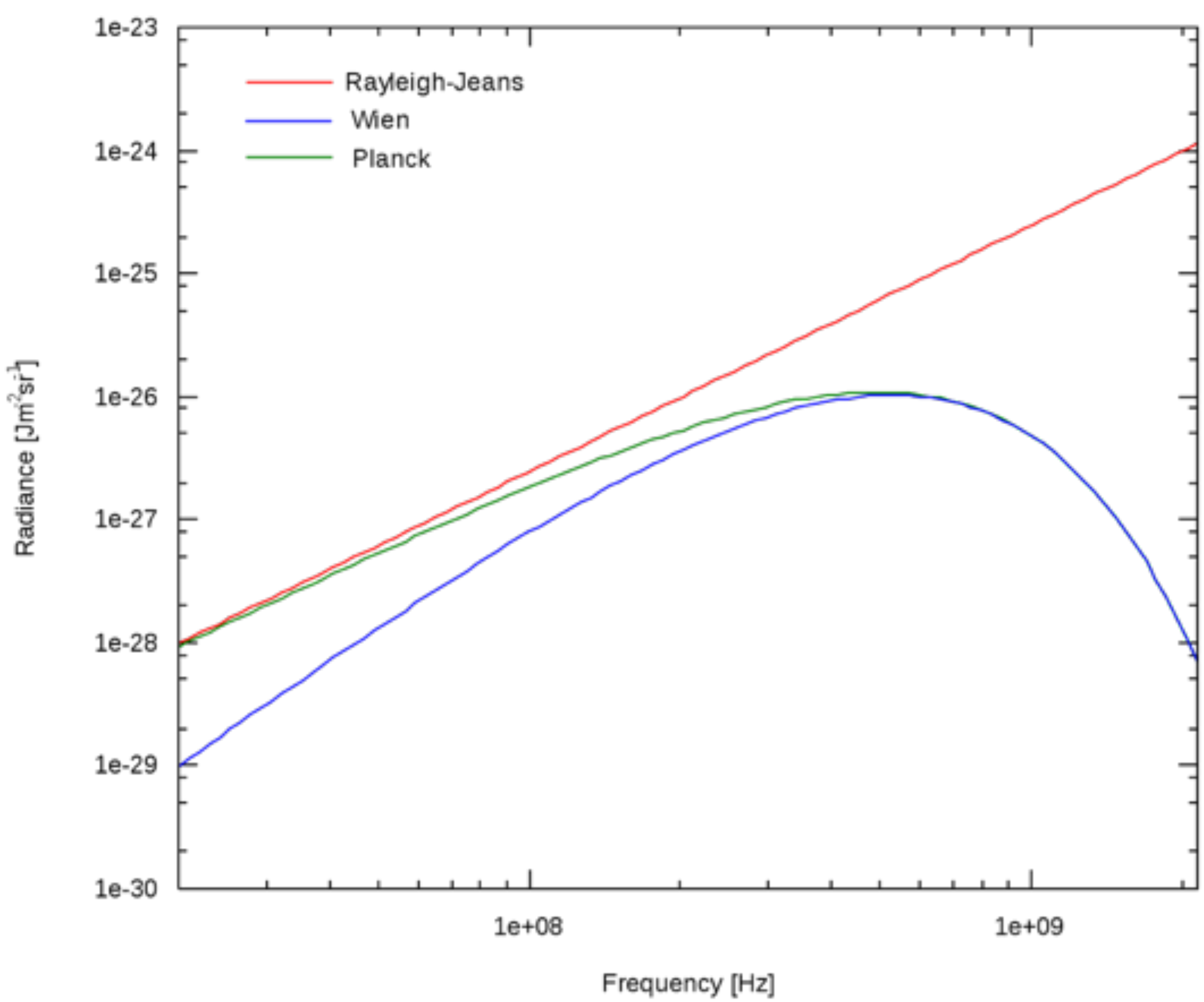
Quantum Universe Center
Inauguration Conference

Outline

- Quantum mechanics & ultraviolet catastrophe
- Quantum gravity
- String/M theory
- M2 & M5 branes as Electric and Magnetic Objects
- Do something about M5 branes
- 6d (1,0) & (2,0) Theories on M5 Branes
- Challenges

Quantum Mechanics

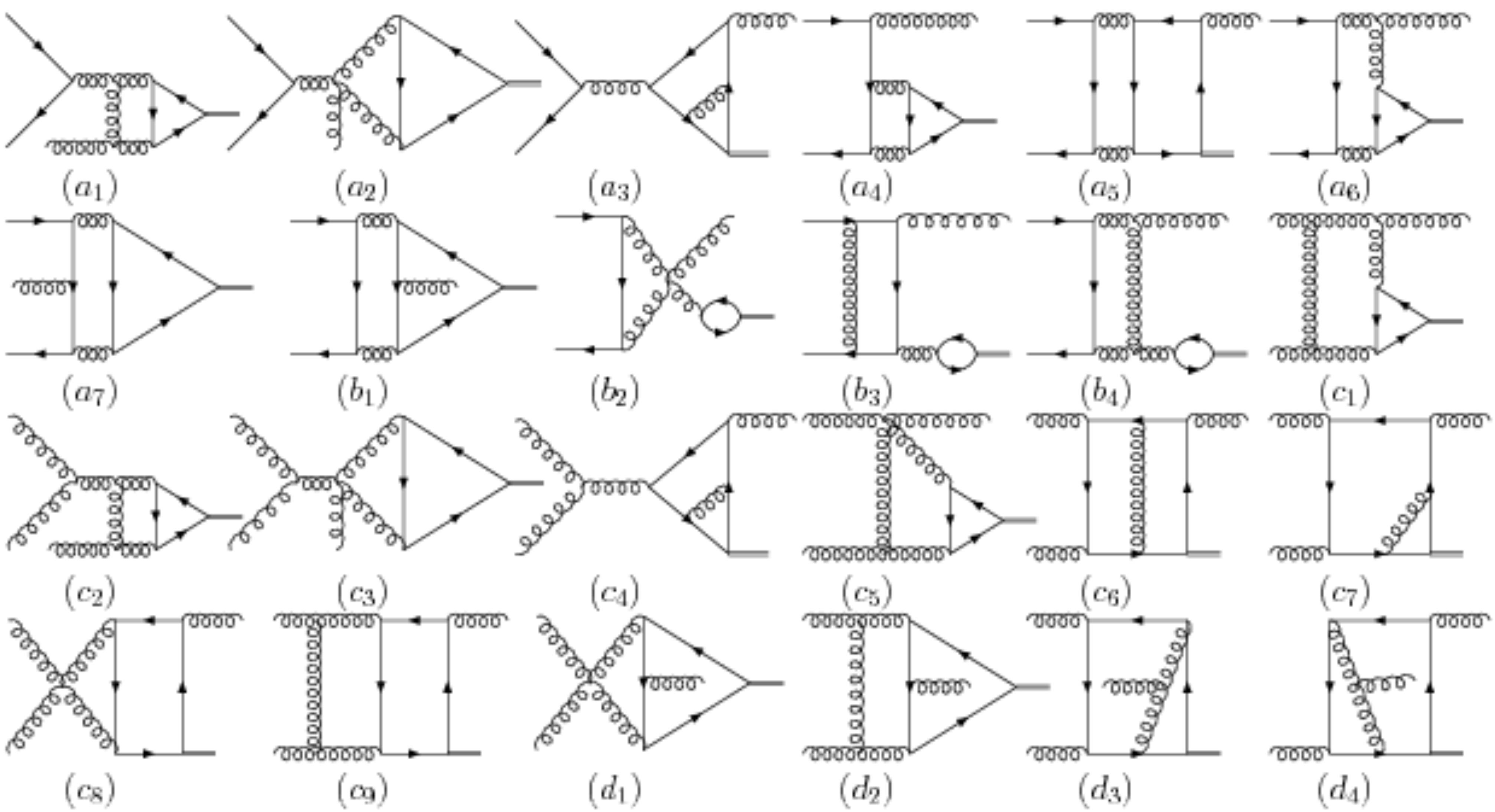
- Planck & Einstein propose quantum of energy
- This cures the ultra-violet catastrophe (infinite degrees of freedom arising from space-time continuum).



Quantum Field Theory

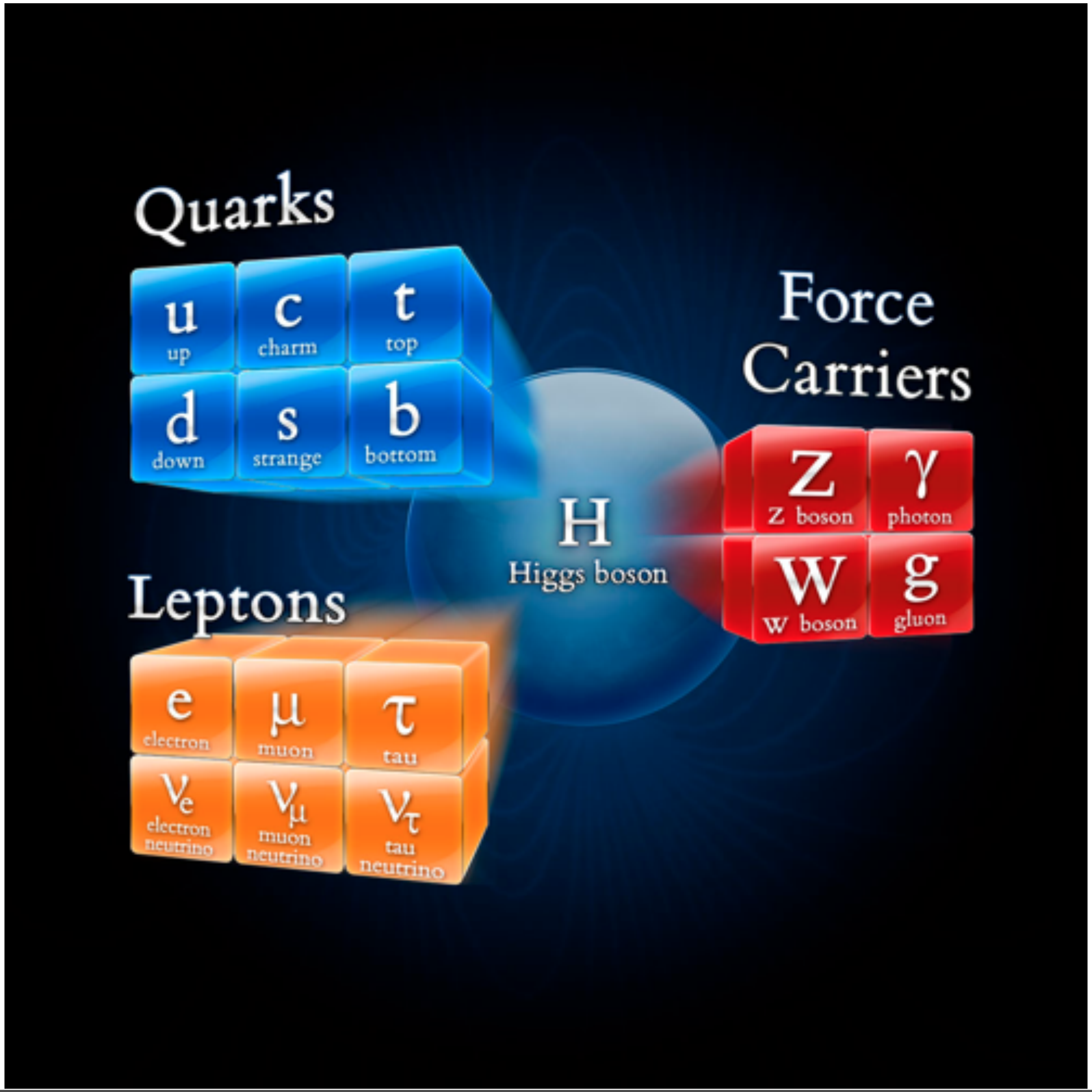
- Identical particles are quanta of a given field
- Lorentz symmetry, locality, causality
- Coupling constant (dimensionless)
- Ultraviolet catastrophe with interactions
- Space-time continuum = infinite number of degrees of freedom
- Intermediate states in the perturbation theory

Quantum Field Theory



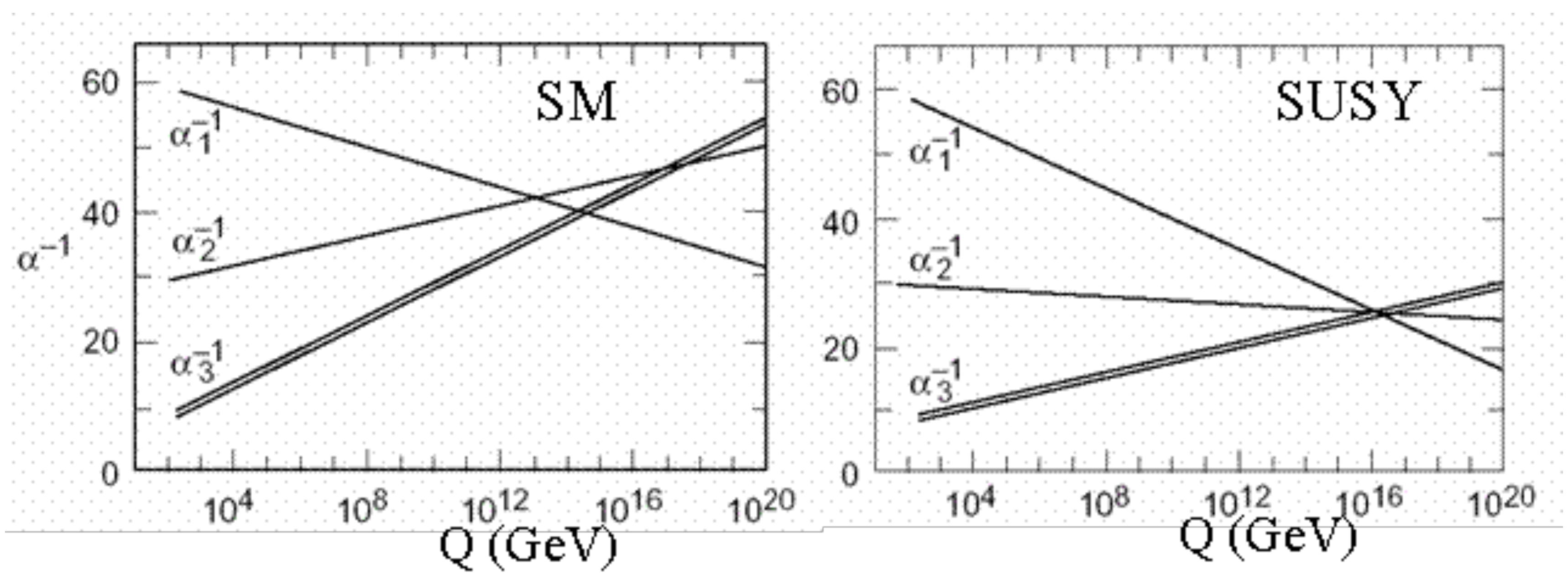
Elementary Particles

- Bosons of integer spin: Bose-Einstein Statistics
- Fermions of half-integer spin: Fermi-Dirac Statistics



Standard Model

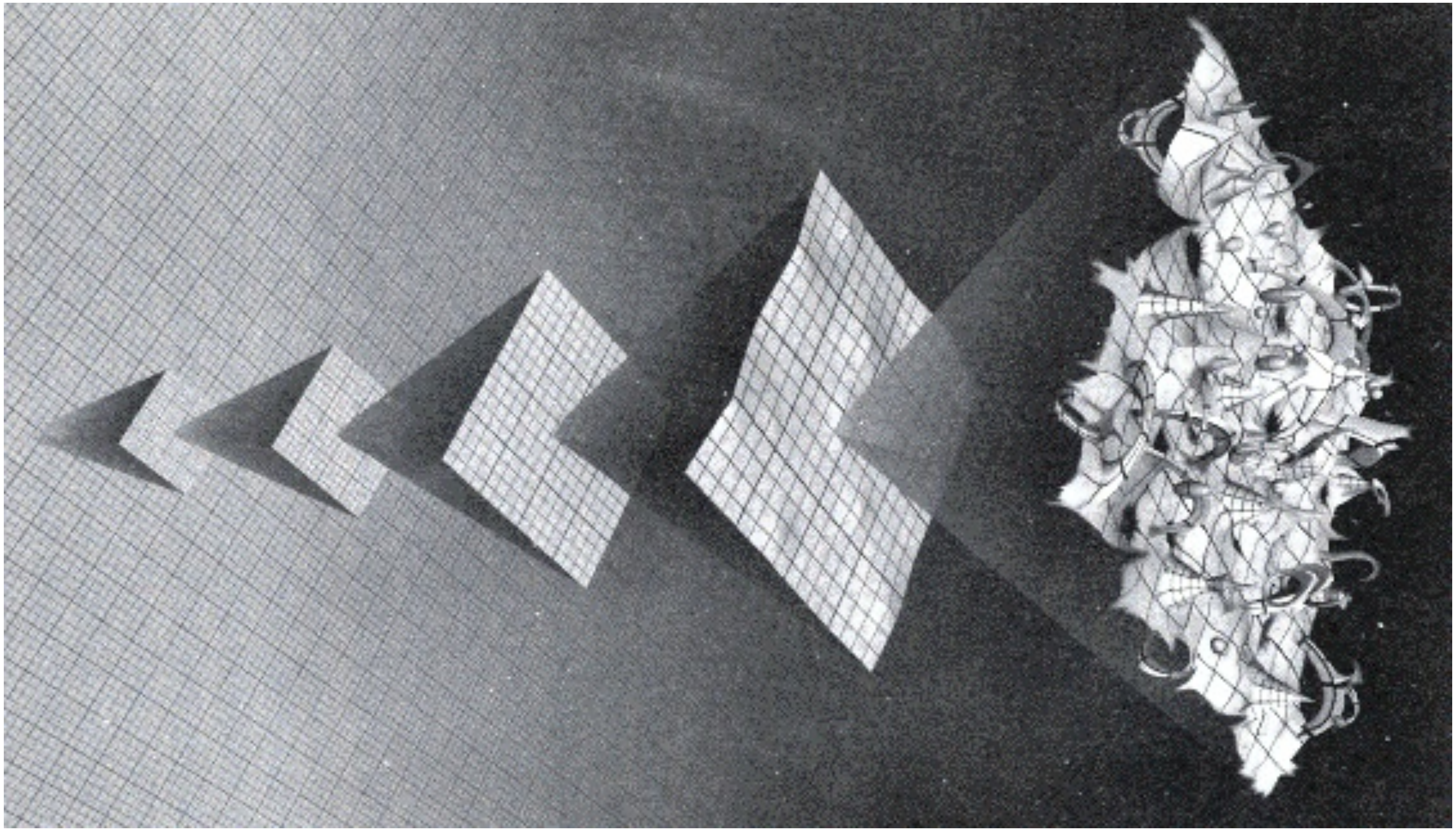
- SU(3)color gauge theory of gluons and quarks
  - The theory of Strong Interaction, Nuclear Force
- SU(2)xU(1) gauge theory + Higgs mechanism for W, Z, photon+ leptons & quarks
  - The theory of Electromagnetic-Weak Interactions
- Running coupling constant: quantum fluctuation

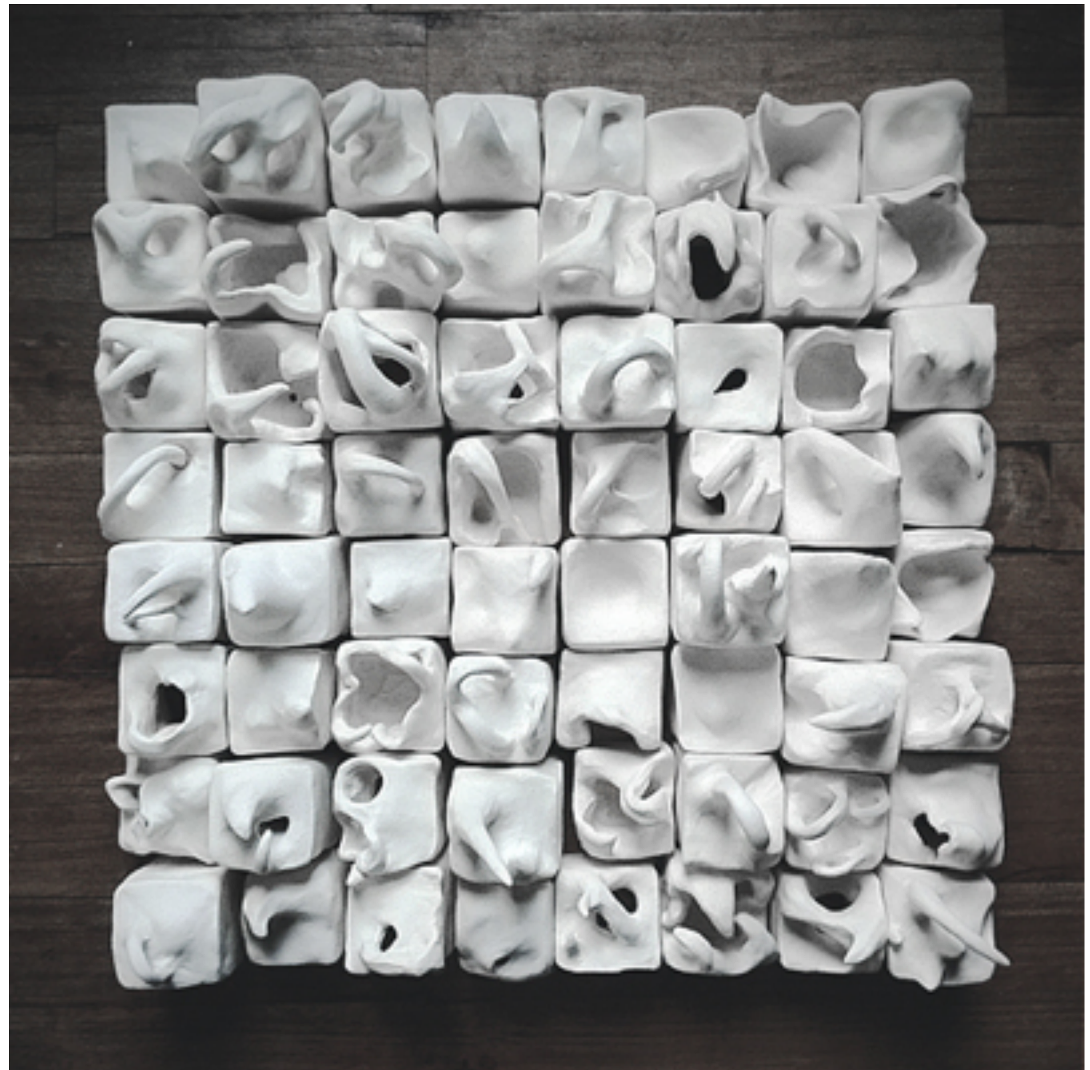


Quantum Fluctuation of Space-Time

- Einstein's general relativity: gravity=curved space-time
  - the equivalence principle: $\exists$ local flat space-time coordinate
- Quantum gravity= quantization of GR
- Beginning of the Universe, Black Holes

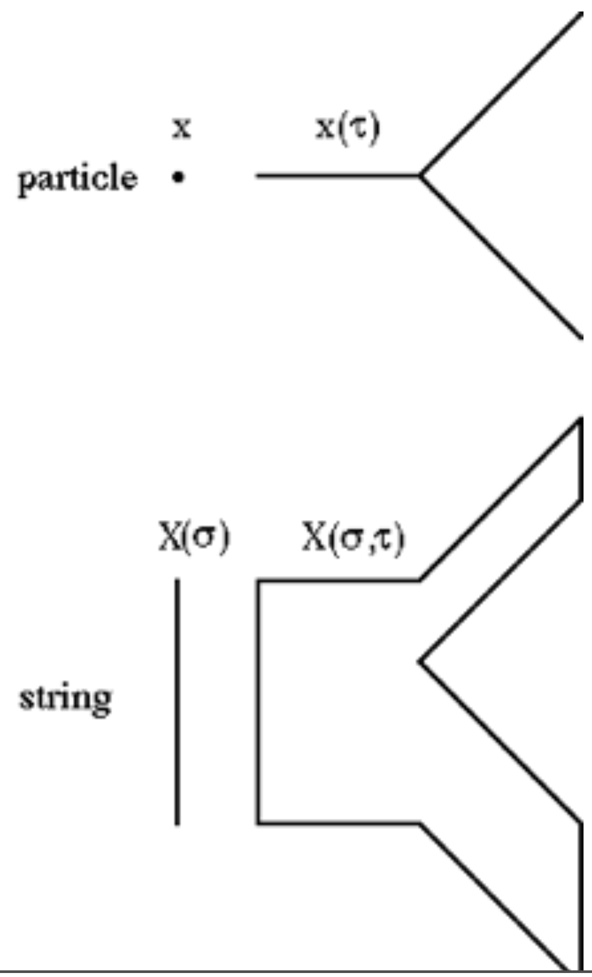
Quantum Fluctuation of Space-Time

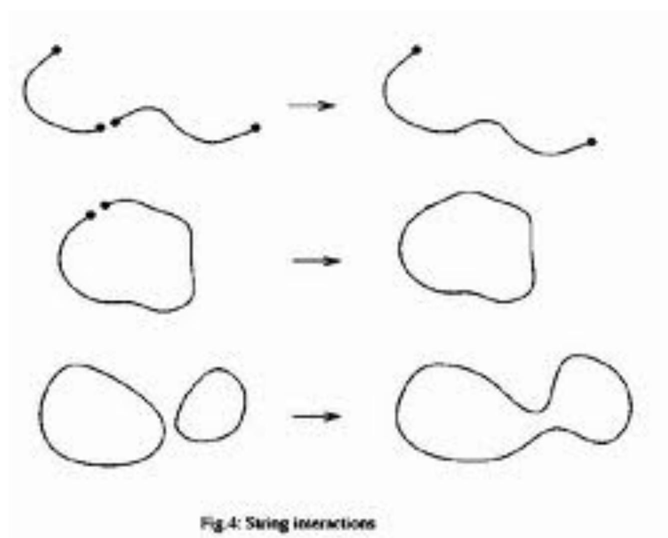




String Theory

- Avoid the space-time local interaction
- consistent ultraviolet behavior
- Nice Vacuum behavior: Supersymmetric Theory in 10-dimensional space-time
- Perturbative string theory: Start with free closed or open





M-Theory

- String theory: □ℓ_s, g_s
- Strong coupling limit of type IIA string theory
  - D0 brane: $g_s \ell_s = R_{11}$, Fundamental string: $\ell_s^2 = L_P^3 / R_{11}$
  - 11-dim M theory on the circle R_{11}
- All 10-dim string theories are related to 11-dim M theory.
- M theory has only one-parameter Planck length L_P: no additional ruler
- Low energy dynamics: 11-supergravity with gravitons, 3-form field, and gravitino

$$G_{MN}, C_{MPQ}, \Psi_M$$

M2 and M5 Branes

- Gauge field: 3-form $C = C_{MNP}\, dx^M \wedge dx^N \wedge dx^P$
- Electrically charged objects: M2 Branes $*d*(dC) = J_3$
- Magnetically charged objects: M5 Branes $*(ddC) = J_6$

$$A_\mu : \ Point\ Charges$$

$$B_{\mu\nu} : Strings$$

$$C_{\mu\nu\rho} : Membranes$$

Challenges of M Theory

- What is the fundamental degrees of freedom? How to write down the theory of M2 and M5 branes? No intrinsic weak coupling limit! No free theory to start with.
- New Fundamental Principle is necessary.
- How to get 4-dim phenomenological model?
  - 4-dim space-time with right kind of symmetry and matter
- Smaller but fundamental problems:
  - the physics on M2 and M5 branes

On N Parallel M2-Branes

- 3-dim superconformal field theory obtained by the infinite coupling limit of 3-dim maximally supersymmetric Yang-Mills theory
- ABJM formalism: 3-dim *N*=6 supersymmetric Chern-Simons matter theory with $U(N)_k x U(N)_{-k}$ gauge group for N M2 branes
- type IIA string theory on AdS4xCP3 space
- 3-dim superconformal field theories
- many contributions to this subject from local community

On N Parallel M2-Branes

$$\begin{aligned}
\mathcal{L}_{\rm CS}+\mathcal{L}_{\rm kin} &= \frac{k}{4\pi}\epsilon^{\mu\nu\rho}{\rm tr}\big(A_\mu\partial_\nu A_\rho - i\frac{2}{3}A_\mu A_\nu A_\rho - \tilde{A}_\mu\partial_\nu\tilde{A}_\rho + i\frac{2}{3}\tilde{A}_\mu\tilde{A}_\nu\tilde{A}_\rho\big) \\
&\quad -{\rm tr}\left(D_\mu\bar{Z}^\alpha D^\mu Z_\alpha - i\bar{\Psi}_\alpha\gamma^\mu D_\mu\Psi^\alpha\right) , \\
\mathcal{L}_{\rm Yukawa} &= -\frac{2\pi i}{k}{\rm tr}\left(\bar{Z}^\alpha Z_\alpha\bar{\Psi}_\beta\Psi^\beta - Z_\alpha\bar{Z}^\alpha\Psi^\beta\bar{\Psi}_\beta + 2\bar{Z}^\alpha\Psi^\beta\bar{\Psi}_\alpha Z_\beta - 2Z_\alpha\bar{\Psi}_\beta\Psi^\alpha\bar{Z}^\beta\right) \\
&\quad -\frac{2\pi i}{k}\epsilon^{\alpha\beta\gamma\delta}{\rm tr}\left(Z_\alpha\bar{\Psi}_\beta Z_\gamma\bar{\Psi}_\delta\right) + \frac{2\pi i}{k}\epsilon_{\alpha\beta\gamma\delta}{\rm tr}\left(\bar{Z}^\alpha\Psi^\beta\bar{Z}^\gamma\Psi^d\right) , \qquad (2.2)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{L}_{\rm potential} &= +\frac{4\pi^2}{3k^2}{\rm tr}\Big(Z_\alpha\bar{Z}^\alpha Z_\beta\bar{Z}^\beta Z_\gamma\bar{Z}^\gamma + \bar{Z}^\alpha Z_\alpha\bar{Z}^\beta Z_\beta\bar{Z}^\gamma Z_\gamma \\
&\quad +4Z_\alpha\bar{Z}^\gamma Z_\beta\bar{Z}^\alpha Z_\gamma\bar{Z}^\beta - 6Z_\alpha\bar{Z}^\alpha Z_\beta\bar{Z}^\gamma Z_\gamma\bar{Z}^\beta\Big) . \qquad (2.3)
\end{aligned}$$

We basically use the convention of [8] except the hermitian gauge fields so that the covariant derivatives now become

$$D_\mu Z_\alpha = \partial_\mu Z_\alpha - iA_\mu Z_\alpha + iZ_\alpha\tilde{A}_\mu , \qquad (2.4)$$

On N Parallel M5 Branes

- 6-dim (2,0) superconformal field theories with ADE type
- 2-form tensor field B, spinor Ψ, scalar Φ_I
- purely quantum (*H=H=dB): self-dual strings
  - $*d*(dB) = *ddB = J_2$
- nonabelian ADE types: N-M5, NM5+OM5, Type IIB on C^2/Γ
- N^3 degrees of freedom
- $AdS_7 \times S^4$ correspondence
- Low energy mode on $R^{1+4} \times S^1$: 5-dim Susy YM with $1/g_{YM}^2 = 4\pi^2/R$
  - instanton = quantum of KK modes of unit momentum

Mysterious M5 Branes

- Many deep implications on 4-dim physics by wrapping M5 on Riemann surfaces with various punctures
- Further implications to lower dimensional physics
- Nonabelian 2-form B and 3-form H=dB fields?
  - Formulation? How to write down theory? Classical action and field equation?
  - tensionless self-dual strings: M2-branes connecting M5 branes
- N^3 Degrees of freedom
  - three string junctions ?

Calculate something on M5

- M-string partition function and DLCQ ✓ Hee-Cheol Kim, Seok Kim, E. Koh, KM, Sungjay Lee: dyonic instantons
- Index function on $S^1 \times S^5$ ✓ Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM [arXiv:1307.7660] The general M5-brane superconformal Index, Hee-Cheol Kim, KM M5 brane theories on R x CP2
- N^3 d.o.f. : ✓ Stefano Bolognesi, KM [arXiv:1105.5073] 1/4 BPS string junctions and N3 problem in 6-dim conformal field theories
- Partition function on S^6 ?
- (1,0) E_8 theory ?
- It is purely quantum theory with unknown Lagrangian.
    - How incomplete is the 5d N=2 Super Yang-Mills theory?

N-Cubic Degrees of Freedom

- Anomaly Coefficient: $C_G = h_G \times d_G / 3$

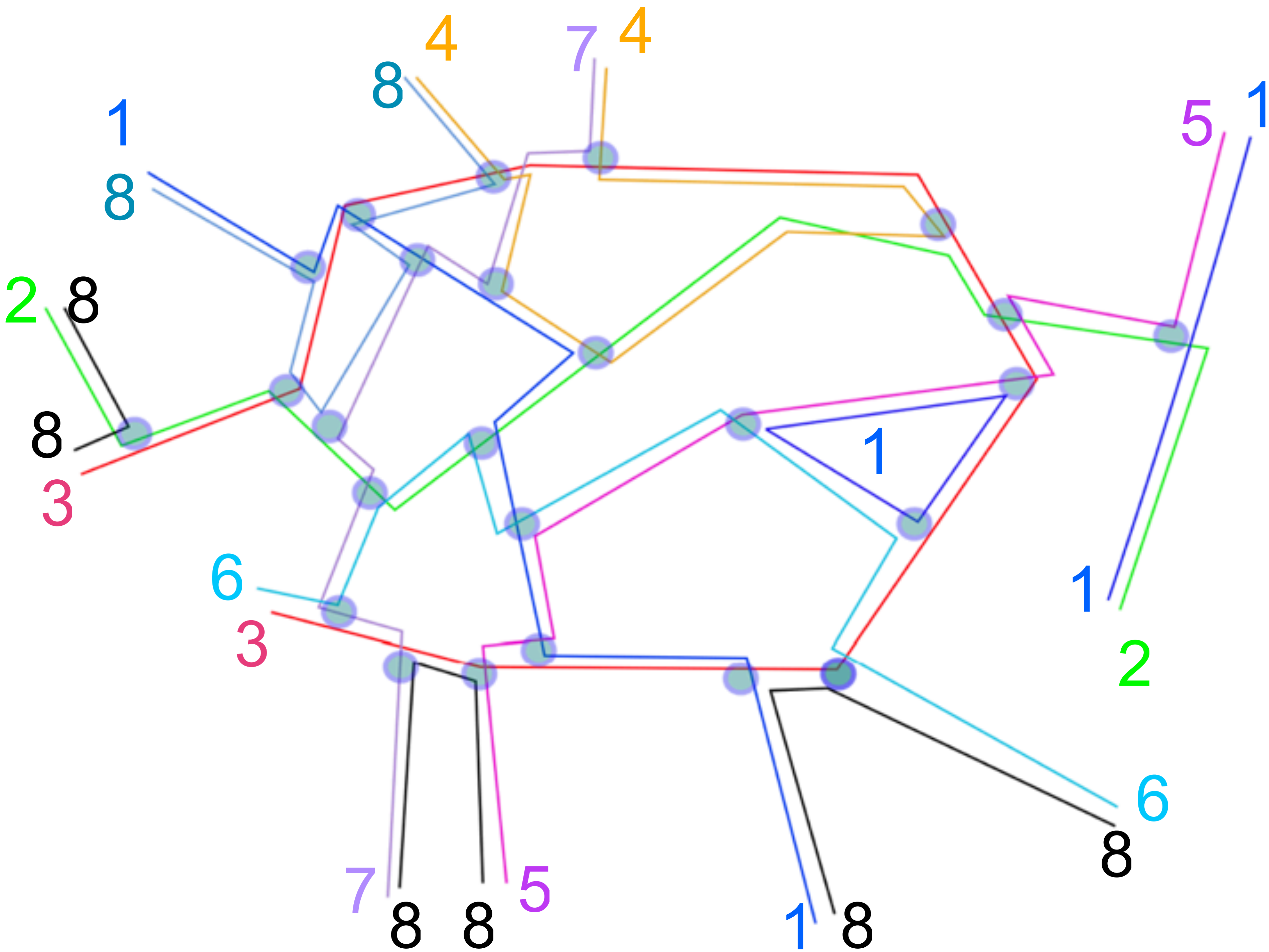
Group	r_G	d_G	h_G	$c_G/3$
$A_{N-1} = SU(N)$	$N-1$	N^2-1	N	$\frac{1}{3}N(N^2-1)$
$D_N = SO(2N)$	N	$N(2N-1)$	$2(N-1)$	$\frac{2}{3}N(2N-1)(N-1)$
E_6	6	78	12	312
E_7	7	133	18	798
E_8	8	248	30	2480

- $C_G = N(N^2-1)/3 = N^2-N \quad + \quad N(N-1)(N-2)/3$

  # of roots,   # of SU(3) root embedding

- Finite temperature phase transition in Coulomb phase
  - beyond Hagedorn temperature, the webs of junctions could dominate the entropy

N-Cubic Degrees of Freedom



Index Function on S^1 x S^5

- Supercharge $Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger$
- BPS bound:

$$E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$$

- Index function:

$$I = \mathrm{Tr}\Big[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta\left(E - \frac{R_1+R_2}{2} - m(R_1-R_2) + aj_1 + bj_2 + cj_3\right)}\Big], \; a + b + c = 0$$

- Euclidean Path Integral of (2,0) Theory on S^1x S^5
  - S^5= S^1 fiber over CP^2 : -i ∂_y = KK modes

$$k \equiv j_1 + j_2 + j_3$$

  - Z_K modding keeps only k/K=integer modes

5d Lagrangian $Q = Q^{++}_{----}, S = Q^{--}_{++++}$

- Lagrangian on R x CP^2 with 2 supersymmetries for any p:

$$S = \frac{K}{4\pi^2}\int_{\mathrm{R\times CP^2}} d^5x\sqrt{|g|}\,\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta - \frac{2i}{3}A_\rho A_\sigma A_\eta\Big)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I + \frac{1}{4}[\phi_I,\phi_J]^2 - 2\phi_I^2 - \frac{1}{2}(M_{IJ}\phi_J)^2 - i(3-p)[\phi_1,\phi_2]\phi_3 - i(3+p)[\phi_4,\phi_5]\phi_3$$
$$-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda - \frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda] - \frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn} + \frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \qquad (2.27)$$

- Supersymmetry Transformation

$$\delta A_\mu = +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda,$$
$$\delta\lambda = +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon.$$

- p/2=-1/2 : $k = j_1+j_2+j_3+R_1+2R_2$
  - additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\ Q^{+-}_{+-+},\ Q^{+-}_{++-}\quad \text{conjugates}$$

the index function on S^1 x S^5

- 5d SYM on S^5 Hee-Cheol Kim, Seok Kim:1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144
  - S-dual version of the index
  - Vacuum energy: $$(\epsilon_0)_{index} = \lim_{\beta'\to 0} \mathrm{Tr}\left[(-1)^F \frac{E-R}{2} e^{-\beta'(E-R_1)}\right] = \frac{N(N^2-1)}{6} + \frac{N}{24}$$
- S^1 x CP^2 path integral off-shell
- Stationary phase: $D^1=D^2=0$, $F= 2s J$, $\varphi + D^3=4s$, $s=\mathrm{diag}(s_1,s_2,\dots,s_N)$
- Path Integral: Off-shell, localization

$$\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^N s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\rm pert} Z^{(1)}_{\rm inst} \cdot Z^{(2)}_{\rm pert} Z^{(2)}_{\rm inst} \cdot Z^{(3)}_{\rm pert} Z^{(3)}_{\rm inst} \,.$$

- For K=1, well-confirmed for k ≤ N with N=1,2,3 with the AdS/CFT calculation

Check with AdS/CFT

- E.g. k = N = 3: (all results multiplied by vacuum energy factor & $e^{-3\beta}$) $y_i = e^{-\beta a_i}$, $y = e^{\beta(m-\frac{1}{2})}$

$$\begin{aligned}
Z_{(2,0,-2)} &= 3\left[y^2(y_1+y_2+y_3)+y(y_1^2+y_2^2+y_3^2)+y^{-1}(y_1+y_2+y_3)-(1+\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots)+y^3\right] \\
&\quad +6y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right]+y^3 \\
Z_{(2,-1,-1)}+Z_{(1,1,-2)} &= -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -4y^3-4y^2(y_1+y_2+y_3)-2y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)+2\left(\frac{y_1}{y_2}+\frac{y_2}{y_3}+\cdots\right)-2y^{-1}(y_1+y_2+y_3) \\
Z_{(1,0,-1)} &= y^3+y^2(y_1+y_2+y_3)-y(y_1^{-1}+y_2^{-1}+y_3^{-1})+1
\end{aligned}$$

add all

$$Z_{SUGRA} = 3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^3-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots\right)+y^{-1}(y_1+y_2+y_3)$$

- ✳ Non-zero flux states contributing to the index
  - ✳ s=(N-1,N-3,…,-(N-1)) = s_0 : SU(N) Weyl vector
  - ✳ index vacuum energy: $E_0 = -\frac{N(N^2-1)}{6}$

Conclusion

- Our world is quantum.
- Our goal is to understand the world.
- One frontier is quantum gravity which dominates the beginning of the Universe and the end of black holes.
- M theory is a potential candidate with very rich and elegant structures. The exploration of this theory is the current goal.
- Some potentially deep insights could be obtained in near future.

Concluding Remarks on QUC Inaugural Conference

- Quantum Universe Center at KIAS is now here not only for KIAS members but also for everybody in our community.
- Through this, we aim for more exchange and exploration of ideas on quantum world and for gaining deeper insights.
- Also, we hope many young people get an opportunity to learn, expand, and form new views on quantum world.
- Thank you all for coming for this audacious moment.